DYNAMICAL LORENTZ SYMMETRY BREAKING FROM *3+1* RENORMALIZABLE MODEL WITH WESS-ZUMINO INTERACTION*

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Abstract: *We study the renormalizable abelian vector-field models in the presence of the Wess-Zumino interaction with the pseudoscalar matter. The renormalizability is achieved by supplementing the standard kinetic term of vector fields with higher derivatives. The appearance of fourth power of momentum in the vector-field propagator leads to the super-renormalizable theory in which the β-function, the vector-field renormalization constant and the anomalous mass dimension are calculated exactly. It is shown that this model has the infrared stable fixed point and its low-energy limit is non-trivial. The modified effective potential for the pseudoscalar matter leads to the occurrence of the quantum dynamical breaking of Lorentz symmetry.*

Anomalous gauge models might be consistently quantized: a first example was provided by the *1+1* dimensional chiral Schwinger model [1]. The situation in the case of *3+1* dimensional chiral gauge theories is a very interesting but still open issue. The basic idea to deal with [2-4], is to restore gauge invariance by means of some additional quantized scalar fields. Here we aim to discuss the *3+1* chiral massless abelian model as described by the lagrangian

$$\mathcal{L}_0[A_\mu,\psi,\bar\psi] = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi\gamma^\mu\left\{i\partial_\mu + eA_\mu P_L\right\}\psi + \text{ gauge fixing },$$

where $P_L \equiv (1/2)(\mathbf{1}-\gamma_5)$ and which leads to the chiral anomaly upon quantization, thereby breaking the classical invariance under local gauge transformations of the left chiral sector.

To restore it, one might attempt to consider the gauge-group extension

$$\begin{aligned}\mathcal{L}_0[A_\mu,\psi,\bar\psi] \longmapsto \mathcal{L}[A_\mu,\psi,\bar\psi,\theta] \quad &= \quad \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar\psi\gamma^\mu\left\{i\partial_\mu + e(A_\mu+\partial_\mu\theta)P_L\right\}\psi \\ &\quad + \text{gauge fixing} + \mathcal{L}_{\text{kin}}[\theta, A_\mu]\ , \qquad (1)\end{aligned}$$

in which the so called (pseudo)scalar Wess-Zumino field indeed appears. Now the low-energy content of the above model is actually described by the effective lagrangian [5]

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_0[A_\mu,\psi,\bar\psi] + \frac{e^3}{48\pi^2}\theta\tilde F_{\mu\nu}F^{\mu\nu} + \dots\ ,$$

where the so called Wess-Zumino interaction arises. It has been shown [6] that models of this kind described by the lagrangian of eq. (1) are indeed, by construction, BRST invariant: however there is a serious conflict between power counting renormalizability vs. perturbative unitarity.

Here, we would like to discuss the properties of the Wess-Zumino interaction, which turn out to be quite relevant with respect to the above issues. The renormalizable abelian vector-field model (in the Euclidean space) we consider is given by the lagrangian which contains the Wess-Zumino interaction and

*Talk given at XI International Workshop on High Energy Physics and Quantum Field Theory, 12-18 September 1996, Sankt-Petersburg

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‡Supported by RFBR (Grant No. 96-01-00535) and by INTAS (Grant No. 93-283-ext).

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¶Supported by INFN grant and by Italian MURST - quota 40/%.

the higher derivative kinetic term:

$$\begin{aligned}\mathcal{L}_{WZ} \quad &= \quad \frac{1}{4M^2}\partial_\rho F_{\mu\nu}\partial_\rho F_{\mu\nu} \; + \; \frac{1}{4}F_{\mu\nu}F_{\mu\nu} \;\; + \; \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \\ &+\frac{1}{2}\partial_\mu\theta\partial_\mu\theta \; + \; i\frac{\kappa}{2M}\theta F_{\mu\nu}\tilde{F}_{\mu\nu} \;, \end{aligned} \tag{2}$$

where $\tilde{F}_{\mu\nu} \equiv (1/2)\epsilon_{\mu\nu\rho\sigma}F_{\rho\sigma}$, some suitable dimensional scale M is introduced, κ and ξ being the dimensionless coupling and gauge fixing parameter respectively.

The Wess-Zumino interaction can be equivalently represented in the following form,

$$\int d^4x \; \frac{\kappa}{2M} \; \theta \; F_{\mu\nu}\tilde{F}_{\mu\nu} \;\; = \;\; - \;\; \int d^4x \, \frac{\kappa}{M} \, \partial_\mu\theta \, A_\nu \tilde{F}_{\mu\nu} \;, \tag{3}$$

when it is treated in the action. Therefore the pseudoscalar field is involved into the dynamics only through its gradient $\partial_\mu\theta(x)$ due to topological triviality of abelian vector fields.

From the above lagrangian it is easy to derive the Feynman rules: namely, the free vector field propagator reads

$$D_{\mu\nu}(p) = -M^2\frac{d_{\mu\nu}(p)}{p^2(p^2+M^2)} + \frac{\xi}{p^2}\frac{p_\mu p_\nu}{p^2} \;, \tag{4}$$

where $d_{\mu\nu}(p) \equiv -\delta_{\mu\nu} + (p_\mu p_\nu/p^2)$ is the transversal projector; the free scalar propagator is the usual $D(p) = (p^2)^{-1}$ and the vector-vector-scalar WZ-vertex turns out to be given by

$$V_{\mu\nu}(p,q,r) = i(\kappa/M)\epsilon_{\mu\nu\rho\sigma}p_\rho q_\sigma \;, \qquad (p+q+r=0) \tag{5}$$

all momenta being incoming, r being referred to the scalar field. It is worthwhile to recall that the Fok space of asymptotic states, in the Minkowskian formulation of the present model, exhibits an indefinite metric structure. As a matter of fact, from the algebraic identity

$$\frac{M^2}{p^2(p^2+M^2)} \;\equiv\; \frac{1}{p^2} \;-\; \frac{1}{p^2+M^2} \;,$$

it appears that negative norm states indeed are generated by the asymptotic transversal component of vector field with ghost-mass M; in addition, the longitudinal components of vector field give rise as well to negative norm states.

Let us now develop the power counting analysis of the superficial degree of divergence within the model. The number of loops is, as usual, $L = I_v + I_s - V + 1$, $I_{v(s)}$ being the number of vector (scalar) internal lines and V the number of vertices. Next we have $2V = 2I_v + E_v$ and $V = 2I_s + E_s$, where $E_{v(s)}$ is the number of vector (scalar) external lines. As a consequence the overall UV behaviour of a graph G is provided by the exponent

$$\omega(G) = \; 4L - 4I_v - 2I_s + 2V - E_s - E_v \; = \; 4 - 2E_v - E_s - 2I_v + 2I_s \;, \tag{6}$$

and therefrom we see that the *only* divergent graph corresponds to $^{\|}$ $I_s = 1, \; I_v = 1, \; E_s = 0, \; E_v = 2$ and it turns out to be the one loop vector self-energy. Thus we conclude that the model is super-renormalizable. We notice that the number of external vector lines has to be even. The computation of the divergent self-energy can be done using dimensional regularization (in 2ω dimensional Euclidean space) and gives

$$\Pi^{(1)}_{\mu\nu}(p) \; = \; \frac{1}{16\epsilon}\frac{\alpha}{\pi}p^2 d_{\mu\nu}(p) + \hat{\Pi}^{(1)}_{\mu\nu}(p) \;, \tag{7}$$

with $\epsilon \equiv 2-\omega$, $\alpha \equiv (\kappa^2/4\pi)$, while the finite part reads

$$\begin{aligned}\hat{\Pi}^{(1)}_{\lambda\nu}(p) \quad &= \quad -\frac{\alpha}{16\pi}p^2 d_{\lambda\nu}(p) \times \left\{ \ln\frac{M^2}{4\pi\mu^2} - \psi(2) + \frac{2}{3} + \frac{2}{3}\frac{p^2+M^2}{p^2}\ln\left(1+\frac{p^2}{M^2}\right)\right. \\ &-\frac{M^2}{3p^2}\left[1 - \frac{p^2+M^2}{p^2}\ln\left(1+\frac{p^2}{M^2}\right)\right] \\ &\left.-\frac{p^2}{3M^2}\left[1 - \frac{p^2+M^2}{p^2}\ln\left(1+\frac{p^2}{M^2}\right) + \ln\frac{p^2}{M^2}\right]\right\} \;. \end{aligned} \tag{8}$$

$^{\|}$Actually the tadpole $E_s = I_v = 1$, $I_s = 0$ indeed vanishes owing to the tensorial structure of the WZ-vertex

where μ denotes as usual the mass parameter in the dimensional regularization. It follows therefore that the single countergraph to be added, in order to make finite the whole set of proper vertices, is provided by the 2-point 1PI structure

$$\Gamma^{(\mathrm{c.t.})}_{\lambda\nu}(p) \equiv - \left.\Pi^{(1)}_{\lambda\nu}(p)\right|_{\mathrm{div}} = -\frac{1}{16}\frac{\alpha}{\pi}p^2 d_{\lambda\nu}(p)\left[\frac{1}{\epsilon} + F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right)\right] , \tag{9}$$

in which F_1 denotes the scheme-dependent finite part (when $\epsilon \to 0$) of the countergraph.

As a result it is clear that we can write the renormalized lagrangian in the form

$$\begin{aligned}
\mathcal{L}^{(\mathrm{ren})}_{WZ} &= \frac{1}{4M_0^2}\partial_\rho F^{(0)}_{\mu\nu}\partial_\rho F^{(0)}_{\mu\nu} + \frac{1}{4}F^{(0)}_{\mu\nu}F^{(0)}_{\mu\nu} + \frac{1}{2\xi_0}(\partial_\mu A^{(0)}_\mu)^2 \\
&\quad + \frac{1}{2}\partial_\mu\theta\partial_\mu\theta + \frac{i\kappa_0}{2M_0}\theta F^{(0)}_{\mu\nu}\tilde{F}^{(0)}_{\mu\nu} \\
&= \frac{1}{4M^2}\partial_\rho F_{\mu\nu}\partial_\rho F_{\mu\nu} + \frac{Z}{4}F_{\mu\nu}F_{\mu\nu} + \frac{1}{2\xi}(\partial_\mu A_\mu)^2 \\
&\quad + \frac{1}{2}\partial_\mu\theta\partial_\mu\theta + i\mu^\epsilon\frac{\kappa}{2M}\theta F_{\mu\nu}\tilde{F}_{\mu\nu} ,
\end{aligned} \tag{10}$$

where the exact, due to super-renormalizability, wave function renormalization constant Z is provided by

$$Z = c_0\left(\alpha, \frac{M}{\mu}; \epsilon\right) + \frac{1}{\epsilon}c_1(\alpha) ; \tag{11}$$

here we can write, up to the one loop approximation,

$$\begin{aligned}
c_0\left(\alpha, \frac{M}{\mu}; \epsilon\right) &= 1 - \frac{\alpha}{16\pi}F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(\alpha^2) , \\
c_1(\alpha) &= \frac{-\alpha}{16\pi} .
\end{aligned} \tag{12}$$

Moreover the relationships between bare and renormalized quantities turn out to be the following,

$$A^{(0)}_\mu = \sqrt{Z}A_\mu , \quad M_0 = \sqrt{Z}M , \quad \xi_0 = Z\xi , \quad \kappa_0 = \mu^\epsilon\frac{\kappa}{\sqrt{Z}} , \quad \alpha_0 = \frac{\alpha}{Z} . \tag{13}$$

In particular, from the Laurent expansion of eq. (13), we can write

$$\kappa_0 = \mu^\epsilon\left\{a_0\left(\kappa, \frac{M}{\mu}; \epsilon\right) + \frac{1}{\epsilon}a_1(\kappa)\right\} , \tag{14}$$

with

$$\begin{aligned}
a_0\left(\kappa, \frac{M}{\mu}; \epsilon\right) &= \kappa + \frac{\kappa^3}{128\pi^2}F_1\left(\epsilon, \frac{M^2}{4\pi\mu^2}\right) + \mathcal{O}(\kappa^5) , \\
a_1(\kappa) &= \frac{\kappa^3}{128\pi^2} .
\end{aligned} \tag{15}$$

This entails that, within this model, we can solve the renormalization group equations (RGE) in the minimal subtraction (MS) scheme $F_1 \equiv 0$: namely,

$$\mu\frac{\partial\kappa}{\partial\mu} = -\epsilon\kappa - a_1(\kappa) + \kappa\frac{d}{d\kappa}a_1(\kappa) , \tag{16}$$

to get the exact MS prescription for β-function

$$\beta(\kappa) = \frac{\kappa^3}{64\pi^2} , \quad \beta(\alpha) = \frac{\alpha^2}{8\pi} , \tag{17}$$

which tells us, as expected, that $\alpha = 0$ is an IR stable fixed point. Therefrom it follows that we can integrate eq. (16) and determine the exact behaviour of running coupling in perturbation theory

$$\alpha(\mu) \;=\; \frac{\alpha(\mu_0)}{1 - [\alpha(\mu_0)/8\pi]\ln(\mu/\mu_0)} \;. \tag{18}$$

Furthermore, from eqs (13) and within the MS prescription, it is straightforward to determine the remaining RG coefficients to be

$$\begin{aligned}
\gamma_M \;&\equiv\; \frac{1}{2}\mu\frac{\partial \ln M^2}{\partial \mu} \;=\; \frac{-\alpha}{16\pi} \;, \\
\gamma_d \;&\equiv\; \frac{1}{2}\mu\frac{\partial \ln Z}{\partial \mu} \;=\; \frac{\alpha}{8\pi} \;, \\
\gamma_\xi \;&\equiv\; \mu\frac{\partial \ln \xi}{\partial \mu} \;=\; \frac{-\alpha}{4\pi} \;.
\end{aligned} \tag{19}$$

In conclusion, we are able to summarize the asymptotic behaviour of the ghost-mass parameter M and of the gauge-fixing parameter ξ at large distances, where perturbation theory is reliable in the model we are considering and within the MS renormalization scheme. Actually, if we set $s \equiv (\mu/\mu_0)$, we can easily derive

$$\begin{aligned}
\bar{\alpha}(s;\alpha) \;&=\; \frac{\alpha}{1-(\alpha/8\pi)\ln s} \;\stackrel{s\to 0}{\sim}\; -\frac{8\pi}{\ln s} \;, \\
\bar{M}(s;M,\alpha) \;&=\; M\sqrt{1-\frac{\alpha}{8\pi}\ln s} \;\stackrel{s\to 0}{\sim}\; M\sqrt{\frac{\alpha|\ln s|}{8\pi}} \;, \\
\bar{\xi}(s;\xi,\alpha) \;&=\; \xi + \ln\left(1-\frac{\alpha\ln s}{8\pi}\right) \;\stackrel{s\to 0}{\sim}\; \xi + 2\ln\left(\frac{\alpha}{4\pi}|\ln s|\right)
\end{aligned} \tag{20}$$

showing that longitudinal as well as ghost-like transversal degrees of freedom of vector fields decouple at small momenta where perturbation theory has to be trusted. Owing to this asymptotic decoupling of negative norm states, within the domain of validity of perturbation theory, the present super-renormalizable model might be referred to as *asymptotically unitary*.

We are ready now to discuss a further very interesting feature of this simple but non trivial model: the occurrence of the radiative Coleman-Weinberg [7] breaking, at the quantum level, of the $SO(4)$-symmetry in the Euclidean version, or the $O(3,1)^{++}$ space-time symmetry in the Minkowskian case. As a matter of fact, we shall see in the following that the effective potential for the pseudoscalar field θ exhibits non trivial true minima and, consequently, some privileged direction has to be fixed by boundary conditions, in order to specify the vacuum of the model. More interesting, those non trivial minima lie within the perturbative domain. Since we are looking for the effective potential of the pseudoscalar field, we are allowed to ignore the renormalization constant $Z(\epsilon)$ and restart from the classical action in four dimensions: namely,

$$\begin{aligned}
\mathcal{A}_{WZ}[A_\mu,\theta] \;&=\; \int d^4x\left\{\frac{\rho}{4M_*^2}\partial_\lambda F_{\mu\nu}(x)\partial_\lambda F_{\mu\nu}(x) + \frac{1}{4}F_{\mu\nu}(x)F_{\mu\nu}(x) + \frac{1}{2\xi}[\partial_\mu A_\mu(x)]^2\right. \\
&\left. +\frac{1}{2}\partial_\mu\theta(x)\partial_\mu\theta(x) + \frac{i}{2M_*}\theta(x)F_{\mu\nu}(x)\tilde{F}_{\mu\nu}(x)\right\} \;,
\end{aligned} \tag{21}$$

in which we introduce the suitable parametrization $M_* \equiv (M/\alpha)$, $\rho \equiv (M_*^2/M^2)$. The generating functional for pseudoscalar background field is defined as

$$\mathcal{Z}[\theta] \;\equiv\; \mathcal{N}^{-1}\int[\mathcal{D}A_\mu]\;\exp\{-\mathcal{A}_{WZ}[A_\mu,\theta]\} \;. \tag{22}$$

The classical field configurations $\bar{A}_\mu(x)$ are the solutions of the Euler-Lagrange equations

$$\frac{\delta\mathcal{A}_{WZ}[A_\mu,\theta]}{\delta A_\mu(x)} \;=\; K_{\mu\nu}[\theta]\bar{A}_\nu(x) \;=\; 0 \;, \tag{23}$$

with ($\triangle \equiv \partial_\mu\partial_\mu$)

$$K_{\mu\nu}[\theta] \;\equiv\; \left(\rho\frac{\triangle}{M_*^2}-1\right)(\delta_{\mu\nu}\triangle-\partial_\mu\partial_\nu)-\;\frac{1}{\xi}\partial_\mu\partial_\nu+\frac{1}{M_*}\epsilon_{\lambda\mu\sigma\nu}\partial_\lambda\theta(x)(-i\partial_\sigma)\;, \tag{24}$$

being an elliptic invertible local differential operator. Therefore, if we set $a_\mu(x)\equiv A_\mu(x)-\bar{A}_\mu(x)$, we eventually obtain

$$\mathcal{Z}[\theta] \;\equiv\; \mathcal{N}^{-1}\exp\left\{-\mathcal{A}_{WZ}[\bar{A}_\mu,\theta]\right\}\times(\det\parallel K_{\mu\nu}[\theta]\parallel)^{-1/2}\;, \tag{25}$$

with $\mathcal{N}=\mathcal{Z}[\theta=0]$.

In order to evaluate the effective potential it is more convenient to consider the dimensionless operator

$$\mathcal{K}_{\mu\nu}[\theta] \;\equiv\; (1/M_*^2)K_{\mu\nu}[\theta]=\;-\top_{\mu\nu}\frac{\triangle}{M_*^2}\left(\rho\frac{\triangle}{M_*^2}-1\right)-\frac{1}{\xi}\frac{\triangle}{M_*^2}\ell_{\mu\nu}\;+\frac{1}{M_*}\epsilon_{\mu\nu\lambda\sigma}\eta_\lambda(x)(-i\partial_\sigma)\;, \tag{26}$$

where we have set

$$\top_{\mu\nu} \;\equiv\; -\delta_{\mu\nu}\;+\;\frac{\partial_\mu\partial_\nu}{\triangle}\;,\quad \ell_{\mu\nu}\;\equiv\;\frac{\partial_\mu\partial_\nu}{\triangle}\;,\quad \eta_\mu(x)\;\equiv\;(1/M_*^2)\partial_\mu\theta(x)\;. \tag{27}$$

We want to evaluate the determinant of eq. (25) for *constant* dimensionless vector η_μ; to this aim we can rewrite the relevant operator into the form

$$\mathcal{K}_{\mu\nu}(\eta) \;\equiv\; \frac{\triangle}{M_*^2}\left(\tau_{\mu\nu}-\frac{1}{\xi}\ell_{\mu\nu}\right)+\mathcal{E}_{\mu\nu}(\eta)\;, \tag{28}$$

with

$$\mathcal{E}_{\mu\nu}(\eta) \;\equiv\; \frac{1}{M_*}\epsilon_{\mu\nu\lambda\sigma}\eta_\lambda(-i\partial_\sigma)\;,\quad \tau_{\mu\nu}\;\equiv\;\top_{\mu\nu}\left(1-\rho\frac{\triangle}{M_*^2}\right)\;. \tag{29}$$

From the conjugation property

$$\left(\mathcal{E}^\dagger\right)_{\mu\nu} \;=\; -\mathcal{E}_{\mu\nu}\;, \tag{30}$$

it follows that

$$\left(\mathcal{K}^\dagger[\eta]\right)_{\mu\nu} \;=\; \left(\mathcal{K}[-\eta]\right)_{\mu\nu}\;, \tag{31}$$

which shows that the the relevant operator is *normal*. As a consequence, after compactification of the Euclidean space, we can safely define its complex power [8] and its determinant [9] by means of the ζ-function technique: namely,

$$\det\|\mathcal{K}[\eta]\| \;=\; \left(\det\left\|\mathcal{K}[\eta]\mathcal{K}^\dagger[\eta]\right\|\right)^{1/2}\equiv\;\exp\left\{-\frac{1}{2}\frac{d}{ds}\zeta_H(s;\eta)\right\}\Bigg|_{s=0}\;, \tag{32}$$

where we have set**

$$(H[\eta])_{\mu\nu} \;\equiv\; (\mathcal{K}[\eta])_{\mu\lambda}\left(\mathcal{K}^\dagger[\eta]\right)_{\lambda\nu}\;, \tag{33}$$

$$\zeta_H(s;\eta) \;\equiv\; \mathrm{Tr}\,(H[\eta])^{-s}\;. \tag{34}$$

After some straightforward calculations, we can definitely obtain

$$\begin{aligned}\mathcal{W}[\eta_\mu,\rho]=-\ln\mathcal{Z}[\eta_\mu,\rho] \;&\equiv\; \mathcal{A}_{WZ}[\bar{A},\eta,\rho]-\mathcal{A}_{WZ}[\bar{A},\eta=\rho=0]\\ &\quad -\frac{1}{4}\frac{d}{ds}\zeta_H(s=0;\eta,\rho)+\frac{1}{4}\frac{d}{ds}\zeta_{h_0}(s=0)\;,\end{aligned} \tag{35}$$

in which

$$\zeta_H(s;\eta,\rho)=2(\mathrm{vol})_4\int\frac{d^4p}{(2\pi)^4}\left\{(p^2)^2\left(1+\rho\frac{p^2}{M_*^2}\right)^2+M_*^2\left((\eta\cdot p)^2-\eta^2p^2\right)\right\}^{-s}\;, \tag{36}$$

**the same regularized determinant is obtained by considering $H'[\eta]\equiv\mathcal{K}^\dagger[\eta]\mathcal{K}[\eta]$.

while, obviously, $\zeta_{h_0}(s) = \zeta_H(s;\eta=\rho=0)$. The effective potential for constant η_μ appears eventually to be expressed as

$$\mathcal{V}_{\rm eff}(\eta,\rho) \ \equiv \ ({\rm vol})_4^{-1}\left\{-\frac{1}{4}\frac{d}{ds}\zeta_H(s=0;\eta,\rho)+\frac{1}{4}\frac{d}{ds}\zeta_{H_0}(s=0)\right\} \ , \tag{37}$$

and therefore we have to compute carefully the integral in eq. (36). To this aim, it is convenient to select a coordinate system in which

$$p_\mu \ = \ ({\bf p},p_4) \ , \quad p_4 \ = \ \frac{\eta\cdot p}{\sqrt{\eta^2}} \ , \tag{38}$$

so that, after rescaling variables to ${\bf v}=({\bf p}/M_*)$, $y=(p_4/M_*)$, we obtain

$$\begin{aligned}({\rm vol})_4^{-1}\zeta_H(s;\eta,\rho) \ &= \ \frac{4}{(2\pi)^4\Gamma(s)}\times\\ \int_0^\infty d\tau\ \tau^{s-1}\int_0^\infty dy\int d^3x\ &\exp\left\{-\tau\left({\bf v}^2+y^2\right)^2\left(1+\rho\left({\bf v}^2+y^2\right)\right)^2+\tau\eta^2{\bf v}^2\right\} \ .\end{aligned} \tag{39}$$

A straightforward calculation leads eventually to the following integral representation [††] [10]:

$$({\rm vol})_4^{-1}\zeta_H(s;\eta,\rho)=\frac{(\eta^2)^{2-2s}}{8\pi^2}\int_0^\infty dt\ \frac{t^{1-2s}}{(1-\rho\eta^2 t)^{2s}}\ {}_2F_1\left(\frac{3}{2},s;2;\frac{-1}{t\,(1-\rho\eta^2 t)^2}\right) \ . \tag{40}$$

Let us first analyze the case $\rho=0$, which corresponds to the low-energy unitary regime; in this limit, the integration in the previous formula can be performed explicitely ($1<{\rm Re}\ s<(7/4)$) to yield

$$({\rm vol})_4^{-1}\zeta_H(s;\eta,\rho=0)=\frac{(\eta^2)^{2-2s}}{16\pi^2\sqrt{\pi}}\frac{2^{4s-4}}{(s-1)}\frac{\Gamma[s-(1/2)]\Gamma[(7/2)-2s]}{\Gamma[(5/2)-s]} \ . \tag{41}$$

In the present case $\rho\to 0$, the effective potential for constant η_μ within the ζ-function regularization is given by

$$\mathcal{V}_{\rm eff}(\eta,\rho=0)=-({\rm vol})_4^{-1}\frac{1}{4}\frac{d}{ds}\zeta_H(s=0;\eta,\rho=0)=\frac{5z^2}{64\pi^2}\left(2\ln z+\frac{7}{15}\right) \ , \tag{42}$$

where $z\equiv(\eta^2/4)$. We see that the stable $O(4)$-degenerate non trivial minima correspond to the symmetry breaking value

$$\ln z_{SB} \ + \ \frac{11}{15} \ = \ 0 \ , \quad z_{SB} \ = \ \exp\{-0.7333\ldots\} \ . \tag{43}$$

We remark that the above result, within the ζ-function regularization, actually reproduces our previous calculation [5] using large momenta cutoff regularization. To be more precise, eq. (42) indeed corresponds to a specific choice of the subtraction terms in the large momenta cutoff method, something that we could call *minimal subtraction for the effective potential*. As a matter of fact we recall that, in general, the ζ-regularized functional determinants of elliptic invertible normal operators are defined up to local polynomials of the background fields.

To sum up, we can draw the following conclusions:
i) in the *3+1* dimensional abelian vector-scalar model with the Wess-Zumino interaction, the renormalization group behaviour allows to reconcile, in some sense, perturbative renormalizability and unitarity, in the asymptotic low momenta domain where perturbation theory is reliable.
ii) It is obviously very interesting to investigate whether a similar feature still holds, within the fully realistic models involving chiral fermions.
iii) Consistent gauge invariant quantization, if any, leads unavoidably to the quantum dynamical breaking of the Lorentz symmetry; this phenomenon has been also noticed [11] in the framework of *2+1* dimensional Chern-Simons theories. The origin of this symmetry breaking is absolutely similar to the Coleman-Weinberg mechanism and is related to the renormalization of one-loop divergences [12]. What a physical meaning could be eventually hidden behind this phenomenon will be clarified elsewhere.

[††] We notice that, from the integral representation (40) for Re $s<1$, it turns out that $\zeta_{H_0}(s)$ is regularized to zero.

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